Halves, Fourths, and Eighths with Geoboards

Materials

- Geoboards with rubber bands (1 per pair)
- Stick-on notes (several per student)
- Transparencies of Student Sheet 2 (3 copies)
- Student Sheet 2 (2–3 per student)
- Student Sheet 3 (1 per student, optional)
- Student Sheet 4 (2 per student)
- Student Sheet 5 (1 per student, homework)
- Scissors (1 per pair)
- Crayons or markers
- Family letter (1 per student)
- Overhead projector

What Happens

Students divide a specified area (the square of a geoboard) into halves, fourths, and eighths. Students discuss their solutions and compare various-shaped fourths to show they are the same size. They write justifications for designs partitioned in fourths. Their work focuses on:

- exploring the relationships among halves, fourths, and eighths
- understanding that equal parts of a whole must be exactly equal in size
- understanding that equal parts of a whole do not need to be congruent
- justifying their division of area into equal parts

Ten-Minute Math: Guess My Number Two or three times during the next few days, do Guess My Number in any spare ten minutes you have outside math class, perhaps at the beginning or the end of the day or before lunch. Pick a number and give enough clues for students to narrow it down to a few numbers. Give students a minute or two to talk with their partners and decide what the number might be. Instead of calling out their answers, students can write down the numbers and hold up their papers for you to see. Write on the board all the students' guesses. Ask if any don't fit or if any are missing. Then give some additional clues to narrow down the guesses to the number you are thinking of. After you provide each clue, ask students which numbers are no longer possibilities and cross them out. In later sessions, instead of providing additional clues, expect the students to ask yes or no questions to narrow down to the exact answer.

You might start with a square number and use the geoboard to illustrate it. **I am thinking of a square number less than 100. It is an odd number. Remember, square numbers are numbers of objects that can be arranged in a square, like the 16 on the geoboard.** (Draw on the board the 4-by-4 square partitioned into 16 small squares and label length and width with 4 and the area inside as 16. Point out that square numbers are also answers when you multiply a number by itself, in this case 4×4.) Ask students what other squares can be made on the geoboard (1×1, 2×2, 3×3). Write their areas as a number series for students to continue: 1, 4, 9, 16, . . .

The square number I am thinking of is an odd number. What number could I be thinking of? Good guesses are 1, 9, 25, 49, and 81. Pick one of these as your answer and give information to narrow it down, such as for 25: **It is one-fourth of a hundred.** Or for 49: **It is a little less than half a hundred.**

In order to prepare students for the 4-by-6 rectangle they partition later in this investigation, use 24 as one of your numbers to guess. Say something like: **I am thinking of a number less than 40 that has more than 5 factors. What are all the numbers it could be?** Good guesses would include 12, 20, 24, 28, 30, 32, and 36. Suggest students narrow down the answer by asking you more about the factors. (The only odd factors are 1 and 3; 5, 7, and 9 are not factors. It is a multiple of 8.)

For full directions and variations of Guess My Number, see p. 55.

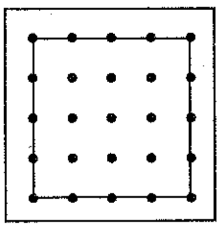
Activity

Halves on the Geoboard and Dot Paper

Introducing Geoboards Pass out the geoboards, one to each pair of students. If your students are using geoboards for the first time, give them some time to explore making shapes before they begin the work on fractions. Most students find geoboards intrinsically interesting and will want just to "mess around" before proceeding on to the structured activities in the unit. (You will probably want to come to an agreement with students about keeping the rubber bands out of the air and on the boards!)

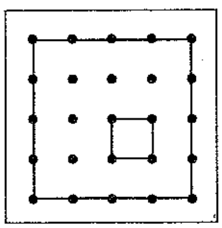
After sufficient exploration time, model for students some ways to use the geoboard to investigate area and fractions. First, show students on the geoboard the whole square they will be dividing into fractional parts. Place a large rubber band around all the outside pins.

This is the largest square you can make on a geoboard. This is the square you will be dividing into fractions when you work with geoboards.

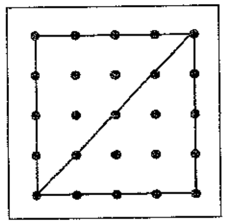


Wrap a rubber band around one of the small squares on the geoboard.

Suppose we count this small square as 1. How many of these small squares are there in the whole geoboard? We're finding its area.



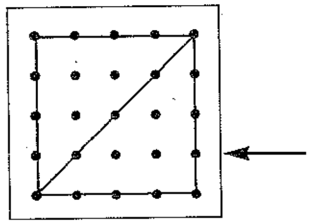
Place a rubber band from corner to diagonal corner on the board.



How large is each of the triangles I made by putting this rubber band on the geoboard?

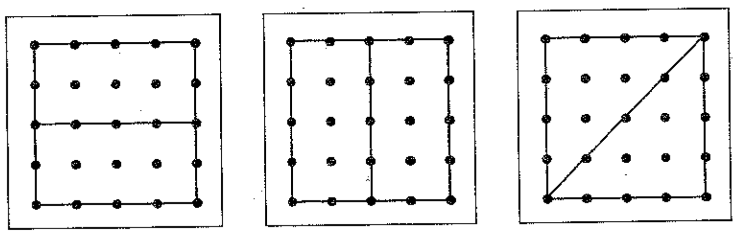
Students are likely to recognize that each piece is a half (as in the crazy cakes activity), and some will know that it therefore has eight square units. Ask them how they know. Point to one of the triangles:

What is the area of this triangle—that is, how many of the little squares are in it?



It is important in the discussion about these shapes that students identify two different ways of knowing that there are 8 squares. Some students will count the area in one triangle, combining the half-unit triangles along the rubber band into squares. Students also should realize logically that there must be 8 small squares on each side of the diagonal because the whole square is 16 small squares, the diagonal divides the square into halves, and half of 16 is 8. Students will need to use both counting unit squares and thinking about relationships in the whole in the next several sessions.

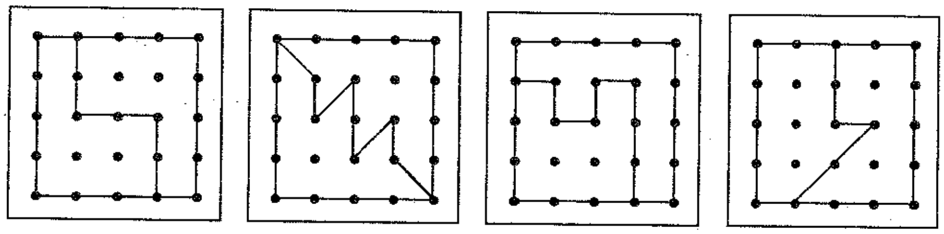
Introducing Dot Paper Pass out Student Sheet 2, Dot-Paper Squares. Note that the squares are similar to the large squares on the geoboards. Draw a few examples of these dot-paper squares on the chalkboard or use an overhead transparency of Student Sheet 2. Ask students what different ways they could divide this square in half. Students will probably first come up with using one horizontal, vertical, or diagonal cut as shown:



If students do not come up with a way to divide the square in half in a more unusual way, ask if a student can suggest an example of one:

These solutions use straight lines to divide the square in half. Can anyone show a way to divide the geoboard into two equal parts using something other than one straight line?

Typically, very little prompting is needed to get students making more interesting halves, and by working together they will get even more ideas. You can draw one of the following to provide an example of using something other than straight lines. The two figures at the right also show that the halves do not need to be the same shape.



For each unusual idea offered by your students, ask them how they know that the two parts are equal in area. When the parts are congruent, encourage explanations that don't involve counting, such as turning one half upside down to show that it is just the same as the other half.

Finding Halves of the Large Squares Using geoboards and recording their solutions on the dot-paper squares (Student Sheet 2), students work in pairs to create as many different ways as they can of dividing the area of the large geoboard square in half. Each student should fill out an individual student sheet.

There are a few rules about making halves that will hold for similar activities in the rest of this unit:

- Each half must be contiguous—that is, it can't have two or more disconnected pieces.
- Each solution must use all the area of the figure. It is against the rules to throw away some of the area and divide what remains into fractional parts.

Share these rules with your students at the beginning of the activity and keep them in mind as you walk around observing their work.

Observing the Students As you circulate among the small groups, observe the strategies students are using, and check to make sure they are reasoning about area (as opposed to pins). Use the following questions to guide your observations:

- How do students divide squares in half? Are they reasoning about the *area* of the squares?

Some students make the mistake of making halves that evenly split the number of *pins* instead of the area. This method often does not result in parts with equal areas. Notice that one of the halves illustrated has three dots inside, some have two, some have one, and one has no dots inside. Suggest to students who are counting pins to think about the crazy cakes examples, where they looked at the whole shape.

Suppose this geoboard was a cake that you wanted to share with a friend. Would it be fair for you to get this piece (point to one of the pieces) and for your friend to get the other piece? How can you prove whether or not it's fair?

Or, you might ask another student to show how to look at the area of the figure instead.

- What do students' halves look like? Do they recognize that equal parts of a whole must be equal in area? Do they realize that equal pieces are not necessarily congruent?
- How do students prove that their squares are divided in half? Do they cut and paste visually? Do they reason with the small unit squares? Do they use knowledge of the whole and parts to find out what's left? Do they have multiple strategies for proving equivalence of areas?

Once students have had a chance to generate some different halves, ask several students to share with the class how their halves are equal in area. You might select one or two students whose halves resulted in shapes that look different. If no one has a shape with parts that are not congruent, then put one on the overhead or chalkboard and challenge the class to prove or disprove that the shape is cut in half. You might challenge pairs of students who seem ready for it to then find halves that do not have the same shape.

If you think your students should have more practice with halves before moving on, ask them to make a page of their favorite halves. Then have them exchange papers to check each other's work.

Activity

Fourths and Eighths

As students are ready to go on to find fourths and eighths, review with them the meaning of *fourths* and *eighths* (for example, one-fourth is "one of four equal parts of the whole"). Students often know the word *fourths*, especially as it fits in the series: thirds, fourths, fifths, sixths, and so on. The fact that the word *quarter* is, in some contexts, a synonym for *fourth* may be less familiar to them.

The last couple of days you have been sharing cakes equally between two people, and using geoboard and dot paper to find halves. What if we shared the cake among four people instead of two? How much cake would each person get?

As students share their ideas, note whether anyone uses "one-fourth" or "one quarter" in his or her discussion. If students do not mention these terms, introduce them now, and show students the notation. Then pose a similar question about eighths:

What if eight people wanted to share the cake equally. How much cake would each person get?

If students do not use the term "one-eighth" to describe such a portion, introduce it now and show the notation for it.

Why do you think that *quarter* might mean the same thing as *fourth*?

Students create a number of examples of both fourths and eighths using Student Sheet 2. If students combine solutions for these two fractions on the same page, ask them to label each square as "fourths" or "eighths." As you circulate among the students, ask them to justify how they know that their division has produced fourths (or eighths):

Is this a "fair" division into fourths (or eighths)? How do you know for sure?

Can you use any of your patterns for fourths to help you create more eighths?

Once students have made some interesting fourths, suggest they find solutions combining different fourths in the same square. Challenge them to find a way to make four fourths in the same square, all different in shape.

See the **Teacher Note**, Students' Work on Fourths and Eighths (p. 16), for some examples.

Many students at this point will have stopped using the geoboard to find divisions of the square and will just draw directly on the grid-paper squares. They will need to work without the geoboard to continue this work at home. At any point in the exploration, however, geoboards are still useful for discussions among students or between you and the students, since the rubber bands can be moved easily to experiment with different configurations.

When students have had some experience dividing squares into fourths and eighths, ask them about quarters and fourths:

Why do you think that *quarter* might mean the same as *fourth?* Why do people say, "We have a quarter of a mile to go" or "It's a quarter after four."

Students are more likely to remember the correspondence between *fourth* and *quarter* if they make the connection between *quarter* as a fraction and *quarter* as a coin that is worth one fourth of a dollar.

You may wish to demonstrate the correspondence between *quarter* and *fourth* on a circular clock by showing how the area swept by the minute hand in a quarter of an hour is one-fourth the area of the face of the clock.

At the end of Session 2, suggest that students continue their work for homework. Pass out additional copies of Student Sheet 2 as needed (see p. 15 for more comments on homework).

Activity

A Page of Favorite Fourths

When students have done a good deal of exploring how to make fourths and eighths, each student creates a page of his or her favorite ways of partitioning the square into fourths. (Since there are fewer options for the shape of individual eighths, partitioning into fourths presents a more interesting task for students.)

While students are working, take around a transparency of Student Sheet 2 to collect single fourths—in this situation, shapes with an area of four units—and to draw a different one on each of the dot squares. Draw the four-unit square and a strip of four on the first two squares and then pick out a variety of others from among the students' favorite fourths. Label each of the shapes with a letter for identification in discussion. If you don't have access to an overhead projector, make a sheet of shapes and copy it for students to look at. See the **Teacher Note**, Equal Fourths? (p. 17), for some examples of single fourths.

As students complete their pages of favorite fourths, they post them so others can look at them now or at any time before you bring the class together to discuss them. As students walk around the room looking at other students' work, suggest that they put stick-on notes with their names or initials on examples they have questions about.

Activity

Equal Fourths?

Bring the class together for students to discuss any of the favorite fourths other students found confusing and to compare fourths you collected on a transparency. This is a chance for students to be "teacher" and present their arguments using the overhead. If you do not have an overhead, students can put the designs on geoboards and hold them up for the class to view. Students using the overhead projector draw on a transparency copy of Student Sheet 2 any shapes they want to discuss.

First, the authors of any controversial partitionings into fourths can explain their thinking. Next, show the students the sheet of fourths you prepared and challenge them to prove the fourths are all the same size.

These are all fourths I copied from your sheets. They are all fourths of the whole geoboard square, so they must all be the same size. Think how

you might prove they are the same size without using any numbers except half. How might you cut and paste this figure to show it is the same size as the strips or the squares that we know are fourths?

Invite students to pick one and go up to the front of the room to explain that it must be a fourth because it is the same as another one already proven. Encourage students to question one another until they see it and encourage them to show different ways to compare a single figure. See the **Teacher Note**, Equal Fourths? (p. 17) for some sample explanations.

Activity

Teacher Checkpoint

Writing Proofs for Fractional Parts

After these two sessions, provide students with designs for fractional parts that they must prove in writing. You can choose designs yourself or use those on Student Sheet 3, Proving Fractional Parts. If you make up your own, choose designs in which the parts are not all the same shape, as in example 3, to see if your students think parts that are not congruent can still have equal areas. Students can explain the fractional parts using words and/or diagrams. Focus on students' ability to describe and/or diagram why and how the parts are equal in area.

Use the following questions to guide your observations of students' work on this activity:

- Are students convinced that the fractional parts are equal? How do they prove it? Do they focus on the area instead of the pins? Do they count squares or parts of squares correctly? Do they use cut-and-paste arguments appropriately? ("If I cut off part of this shape and paste it on over here, then they would be exactly the same shape.")
- How do students use words and/or diagrams to prove that fractional parts are the same? Are their proofs convincing? Can you follow students' reasoning?

See the **Teacher Note**, Students Write About Fractions (p. 18), for some samples of the sorts of arguments and diagrams your students might use in proving the equality of fractional parts.

Activity

Making a Fourths Quilt

Give each student two copies of Student Sheet 4, Squares for a Quilt of Fourths, one for doing a draft version and one for doing a final version of a quilt pattern. Students divide each of the small squares into fourths in a different way. They use their favorite fourths or make up new ways of partitioning into fourths. They color each square's fourths using the same four colors. It is easier for students to do this if they color each square right after they divide it into four parts. Students may take these home to finish. The quilts can be cut out from the student sheet and displayed along the top edge of a wall to form a frieze.

Sessions 2, 3, and 4 Follow-Up

Homework

Dot-Paper Squares After Session 2, send home the family letter or *Investigations* at Home. Students will also need Student Sheet 2, Dot-Paper Squares. For homework, students should finish dividing the squares into fourths and eighths or work on their page of favorite fourths, depending on how far they have progressed in class.

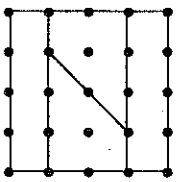
Dividing into Fourths After Session 3, students continue working at home on favorite fourths.

A Favorite Fourth After Session 4, students choose one of their favorite fourths, one they think is the most interesting, and record it on Student Sheet 5, A Favorite Fourth. For homework, students write about how they would prove that each part is one-fourth and that all four parts are equal.

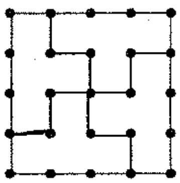
Teacher Note

Students' Work on Fourths and Eighths

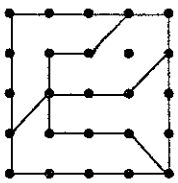
Students are likely to see quickly some of the common ways of dividing a square into fourths and then into eighths, but they will have to think harder about more irregular ways. Here are some:



Rashaida's:
Some different

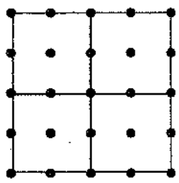


Luisa's :
All congruent

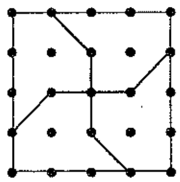


B.J.'s: All different

One interesting way to generate fourths is to start with a simple division into fourths, such as example A, and to cut a small piece from each fourth to add to the next fourth, as in example B. As long as you do the same thing to all four fourths, the resulting figure will also be divided into equal fourths. This is a fairly sophisticated argument for students to understand, but if you show them an example, they are likely to use it to generate more examples.

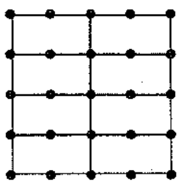


A

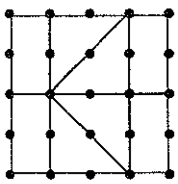


B

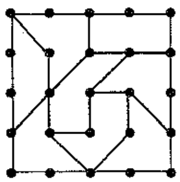
When working with eighths, students will likely first come up with a picture like Kim's, below. Others may then modify it to something like Jesse's. More unusual eighths, like Qi Sun's, may emerge more slowly, as these eighths require a deeper understanding.



Kim



Jesse

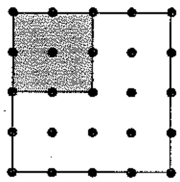


Qi Sun

Equal Fourths?

tion and *quarter* as a coin that is worth one fourth of a dollar.

You may wish to demonstrate the correspon-



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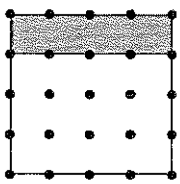

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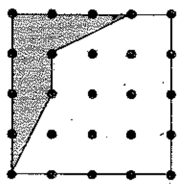

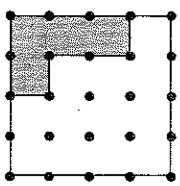

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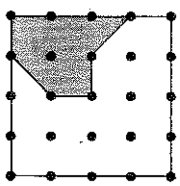

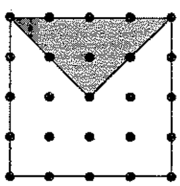

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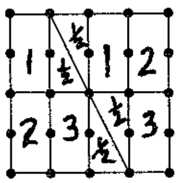
Teacher Note **_Students Write About Fractions_**

projector, make a sheet of shapes and copy it for students to look at. See the **Teacher Note**, Equal Fourths? (p. 17), for some examples of single fourths.

As students complete their pages of favorite fourths, they post them so others can look at them now or at any time before you bring the class together to discuss them. As students walk around the room looking at other students' work, suggest that they put stick-on notes with their names or initials on



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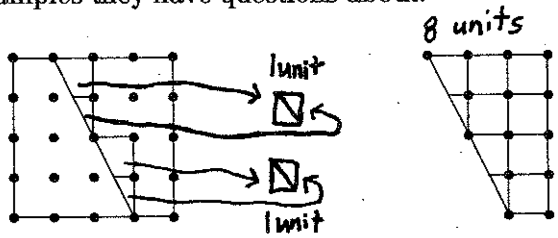

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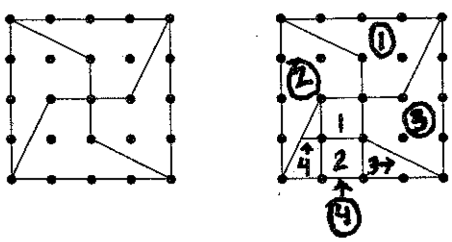

fourths can explain their thinking. Next, show the students the sheet of fourths you prepared and challenge them to prove the fourths are all the same size.

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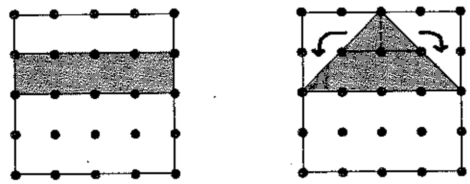


A B

Think how you might prove they are the same size without using any numbers except half. How might you cut and paste this figure to show it is the same size as the strips or the squares that we know are fourths?

Invite students to pick one and go up to the front of the room to explain that it must be a fourth because it is the same as another one already proven. Encourage students to question one another until they see it and encourage them to show different ways to compare a single figure. See the **Teacher Note**, Equal Fourths? (p. 17) for some sample explanations.

After these two sessions, provide students with designs for fractional parts that they must prove in writing. You can choose designs yourself or use those on Student Sheet 3, Proving Fractional Parts. If you make up your own, choose designs in which the parts are not all the same shape, as in example 3, to see if your students think parts that are not congruent can still have equal areas. Students can explain the fractional parts using words and/or diagrams. Focus on students' abil-

Sessions 1 and 2

Thirds, Sixths, and Twelfths

Materials

- Rulers (1 per pair)
- Crayons or markers
- Student Sheet 7 (1 per pair)
- Transparency of Student Sheet 7
- Student Sheet 8 (4 per student)
- Student Sheet 9 (1 per student, homework)
- Geoboard (1 for teacher demonstration)
- Calculators (1 per pair, Ten-Minute Math)
- Small stick-on notes (several per student)
- Overhead projector

What Happens

Students use a larger whole to explore size and equivalence relationships among fractions based on one-third. They divide dot-paper rectangles (as opposed to squares) into thirds, sixths, and, optionally, twelfths. They discuss that a fraction of these rectangles is larger than the same fraction of the squares with which they have been working. Each student makes a page of favorite sixths to keep. Their work focuses on:

- exploring the relationships among thirds, sixths, and twelfths
- understanding that the same fraction of different-sized wholes will be different in area
- being able to justify their division of rectangles into thirds, sixths, and twelfths

Ten-Minute Math: Guess My Number Continue to do Guess My Number outside of math class in any spare five or ten minutes you have. Pick a larger number, for example: **It is a three-digit number. It is a multiple of fifteen. Two of its digits are the same.** Students may use calculators to find the multiples of fifteen. (It could be 225, 300, 600, 900.) Suggest students narrow it down by asking about the digits in the number—for example, does it have a 3?

When students are comfortable with the activity, give them less information and expect them to narrow down the number by asking you questions to which you can answer yes or no. Limit the number of questions they can ask about the size of the number (Is it bigger than 500? Is it between 200 and 300?). Suggest that they ask you questions about its factors (Is it odd? Is it a multiple of ____?). For variations of Guess My Number, see p. 56.

Activity

Working with Thirds and Sixths

Put a transparency of Student Sheet 7, A Dot-Paper Square and Rectangle, on the overhead and pass out 1 copy of Student Sheet 7 to each pair of students. Also distribute one copy of Student Sheet 8 to each student.

Examining Fractions of Different Wholes Before you introduce thirds and sixths, your students need to consider the new dot paper they will be working with. The fact that the dot-paper rectangles on Student Sheet 8

have a different area from the dot-paper square on Student Sheet 7 can be confusing for your students. A potentially confusing fact, for example, is that half of the rectangle (12 small squares) is larger than half of the square (8 small squares). A third of the rectangle is the same area as half of the square—both are 8 small squares in area. In order to have students focus on the fact that *equal fractions of different-sized wholes are different,* display the transparency of Student Sheet 7 for all students to see.

Point to the dot-paper square as you review with students the total number of small squares in it. Compare it to the total number of small squares in the dot-paper rectangle on the transparency.

Pretend that each of these is a cake. You can choose one to share fairly with a friend. Which one would you choose? Why?

Students discuss this question in pairs or small groups before sharing their ideas with the whole class. Encourage students to reason about the size or area of the halves.

Do you think that half of the rectangle will have more or fewer small squares than half of the square? Why?

Then ask students to figure it out exactly. Draw a diagonal on the rectangle on the overhead and label it *Halves.*

How many squares are in half of this rectangle? How does half of the rectangle compare to half of the square?

Even after students have discussed the new whole, it is important that you listen for students still thinking that eight squares is equal to one-half, no matter what the whole. If this kind of thinking is widespread, reiterate the points above about the differences in area between the dot square and the dot rectangle.

Finding Thirds Begin by asking students about a sharing situation involving three people.

We have been sharing things, like cake, among 2 people, 4 people, and 8 people—finding halves, fourths, and eighths. What if we shared a cake among 3 people instead of two? How much cake would each person get?

If anyone uses "one-third" as a way to describe this portion, ask that student to explain what "one-third" means. If students do not know about thirds, explain that they are three equal parts. If you want to share a cake equally among three people, you divide it into thirds.

Tell students that they will now be looking for thirds. Distribute Student Sheet 8, Dot-Paper Rectangles.

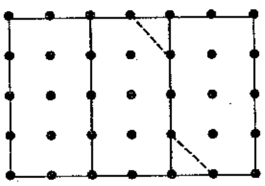
How can you divide one of the rectangles on Student Sheet 8 into thirds?

After each student has done this, have two or three present different solutions. You may choose to have students draw their solutions on either the board or the overhead. Ask the class:

How can we be sure each rectangle is divided into thirds?

How many small squares are in one-third of the rectangle? That many squares was what fraction of the square?

Students may find working with thirds difficult, even if they have been successful with halves, fourths, and eighths. Halves, fourths, and eighths are easier to understand because they can all be created by halving over and over (for example, a fourth is half of a half). Understanding thirds requires understanding a different way to divide a whole. Some students jump to the conclusion that thirds are three units, perhaps because fourths were four units in the square. Review the definition of *fourths* as four equal parts that take up the whole area and ask students what the definition of *thirds* must be. They need to understand that thirds (and sixths) of one whole, like fourths, must be exactly equal in area. They may not look the same, but they must have the same number of square units.



Generating Examples Students now work in pairs to create a page of thirds and a page of sixths. Once they have made simple thirds, they can make new thirds. As they may have done with fourths, some students might take a small piece from one third and add it to another third, thus making interesting thirds. Some students will make sixths by halving their thirds. For a strategy a student invented, see the **Teacher Note**, A Strategy for Finding Interesting Thirds (p. 28).

Students can work on their own if they choose, but they must verify their solutions with another student. Circulate among students and explore how they are thinking about this activity. They should be able to justify their responses as before. Challenge students to come up with "unusual" thirds and thirds that don't all have the same shape, as they did before with halves and fourths. Expect that some students will argue by counting units of area, while others will "see" the equivalence between two areas. As with partitioning areas of squares, be sure students count area, not dots.

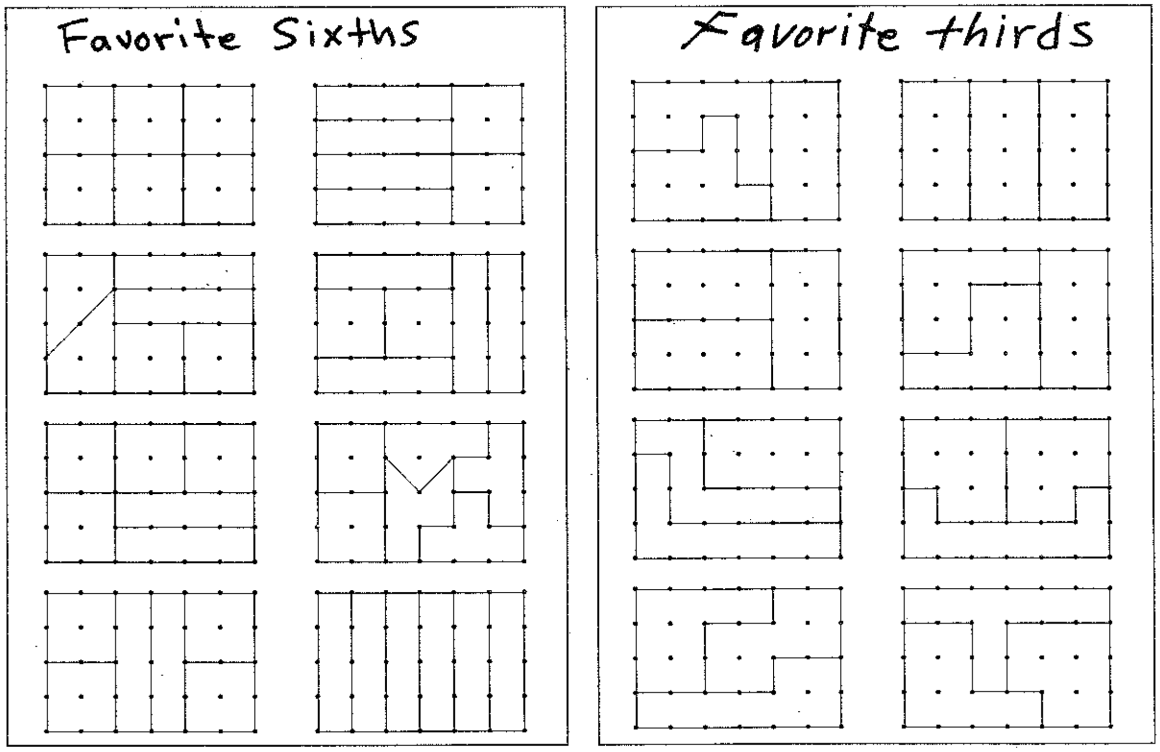
You can present the task of dividing the rectangle into *sixths* at the same time as the introduction of thirds, or you may visit students as they work on thirds and suggest they go on to sixths when you think they've done sufficient work on thirds. In either case, encourage students to use their prior solutions for halves and thirds to help them think of more possibilities for sixths. Some students may also want to investigate twelfths.

Activity

A Page of Favorite Thirds and Sixths

Once students have finished making thirds and sixths in their pairs, they make final copies of their favorite solutions for thirds and sixths, with both fractions on one page or a page of each. It is fine if pairs of students who have collaborated use similar examples to make their individual pages. The important criterion is that students need to be confident the solutions they choose in fact show sixths accurately and that they can prove this to another person.

As with favorite fourths, you might post students' pages of favorites so they can put stick-on notes on ones they don't understand. Or you can collect from students a variety of partitionings into thirds and sixths on an overhead transparency for students to discuss.



Sessions 1 and 2 Follow-Up

Homework

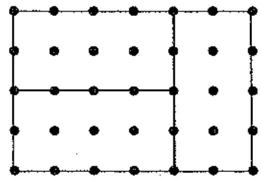
Dot-Paper Rectangles After Session 1, students may complete their pages of thirds or sixths or make a page of favorites.

Thirds and Sixths After Session 2, give each student a copy of Student Sheet 9, Thirds and Sixths. Students divide one rectangle into thirds and one into sixths and explain how they know each rectangle is divided into equal parts.

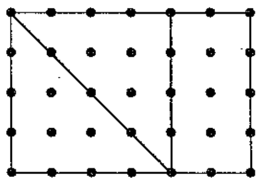
Teacher Note

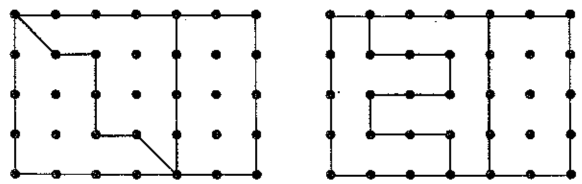
A Strategy for Finding Interesting Thirds

While making thirds and sixths may be a difficult task, it leads students to construct firmer ideas about the relationships among sixths and thirds and about the nature of fractional parts. During this work, listen for interesting discoveries or strategies that students invent. You may want to share these with the whole class. For example, one student who was working on thirds started with this pattern:



He then invented the following strategy: "If you cover up the rectangle on the (right) side, then you have the squares, like we used before, so just do halves. You can use all your old halves ways, then uncover the other rectangle and you have thirds!"

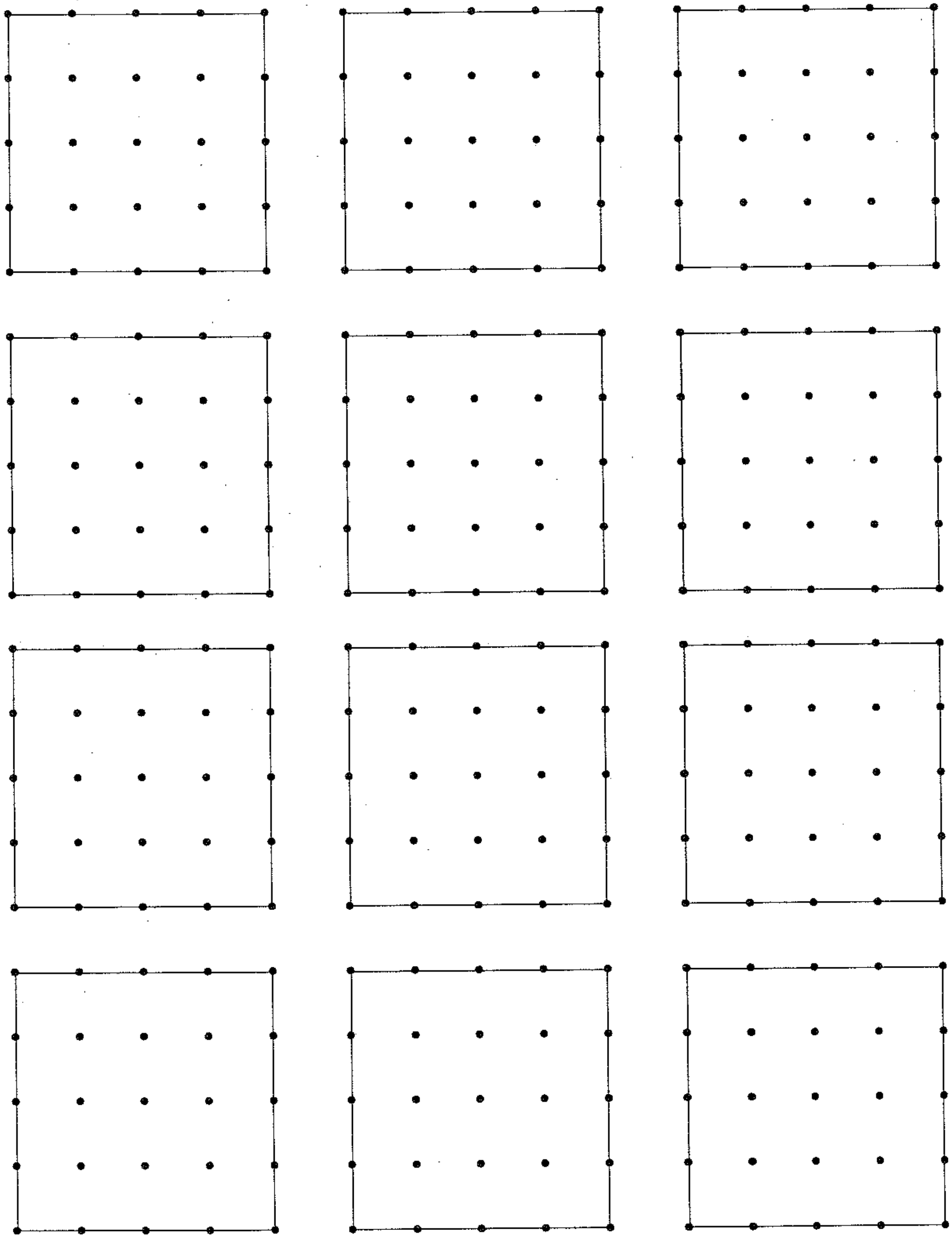




Encourage your students to come up with similarly inventive methods to create examples of thirds or sixths as they work through the activities in this investigation.

Name Date

Dot-Paper Squares

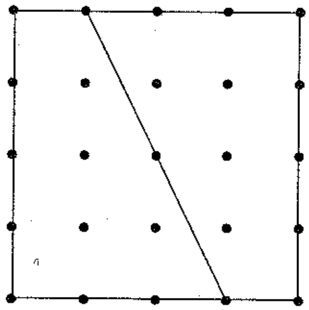


Name Date

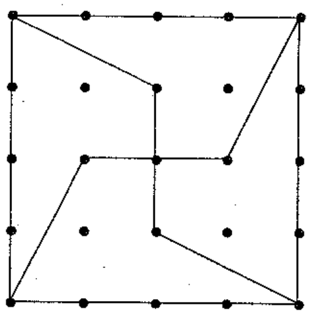
Student Sheet 3

Proving Fractional Parts

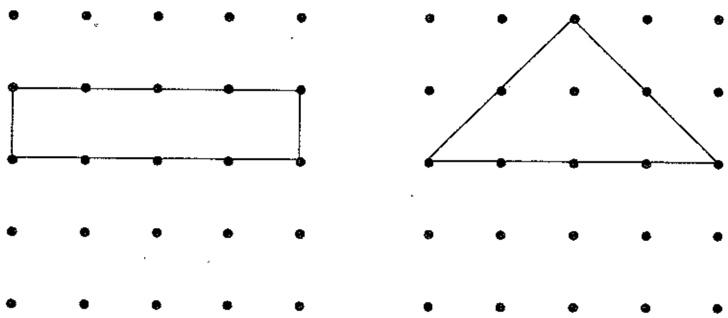
1. Prove that this square is divided into halves.



2. Prove that this square is divided into fourths.



3. Prove that these two shapes have equal area.

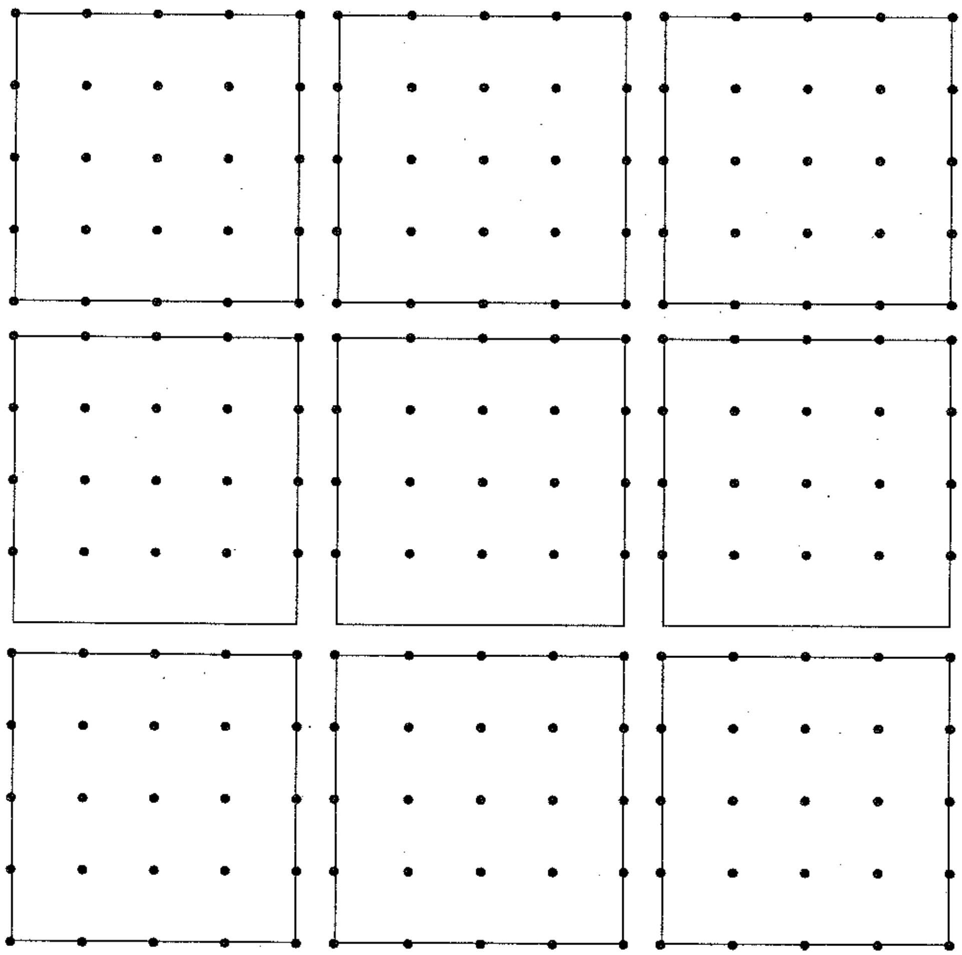


Name Date

Student Sheet 4

Squares for a Quilt of Fourths

Divide each small square into fourths in a different way. Use your favorite fourths, or make up new ways of dividing into fourths. Color each square's fourths right after you divide it into four parts. Use the same four colors.



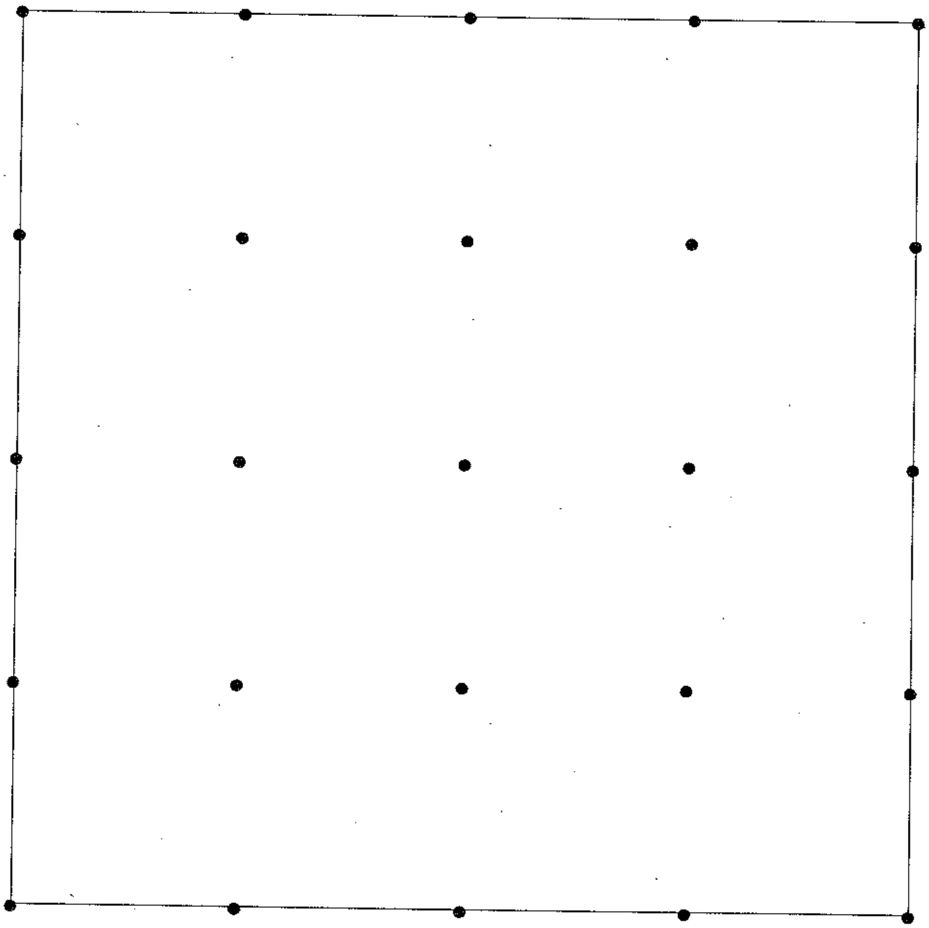
Name

Date

Student Sheet 5

A Favorite Fourth

Draw one of your favorite or most interesting fourths.

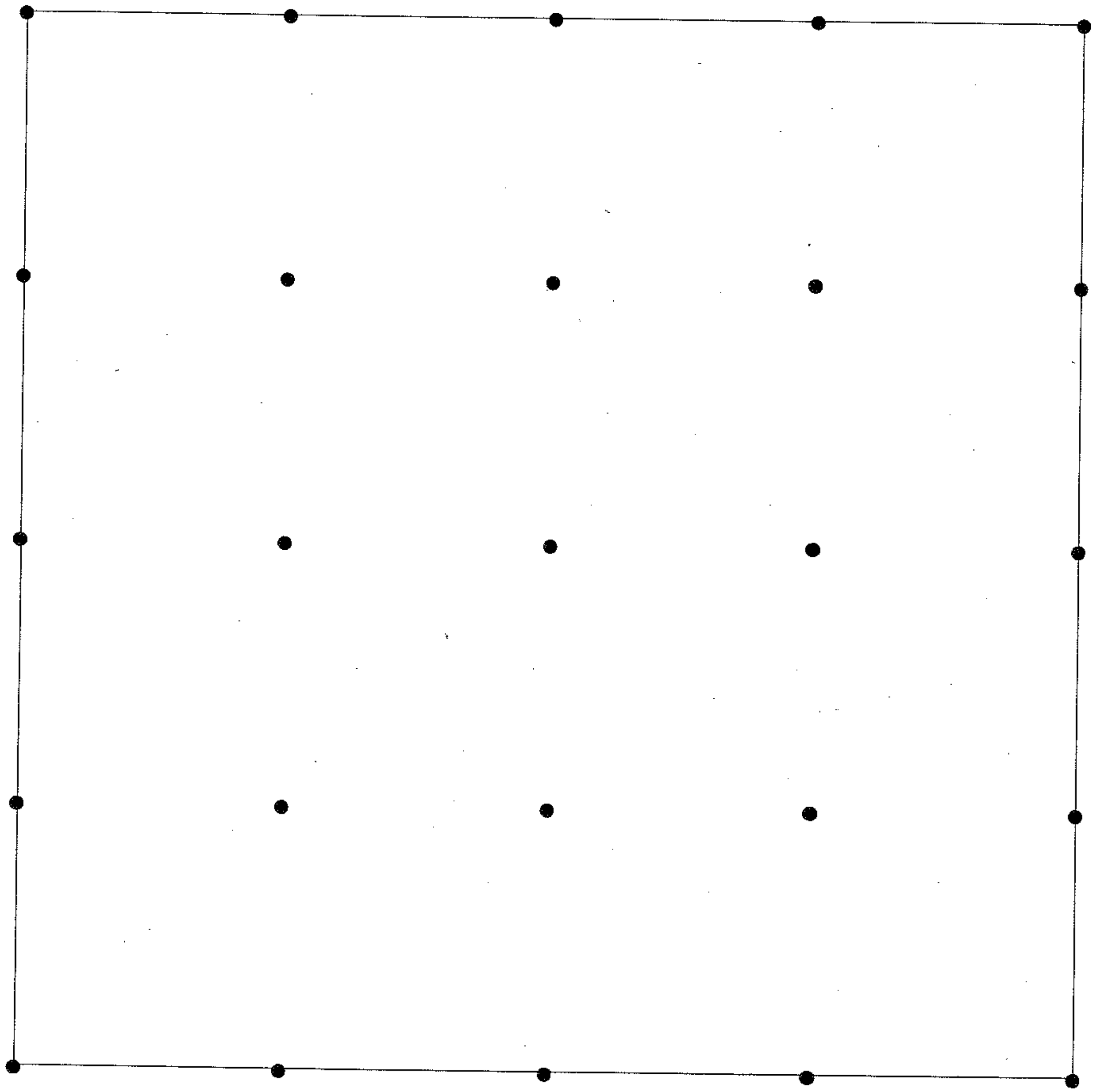


Use diagrams and/or words to prove that this square is divided into fourths. Explain how you know that each piece is one-fourth.

Name Date

Student Sheet 6

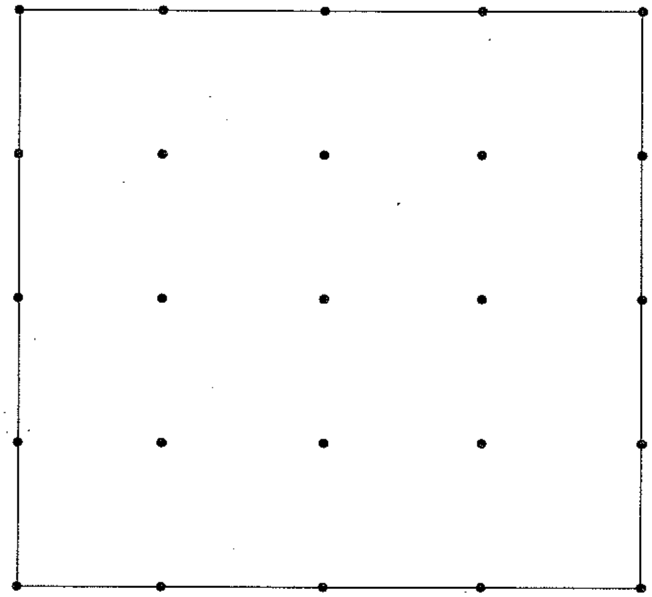
Large Dot Square for Combining Fractions

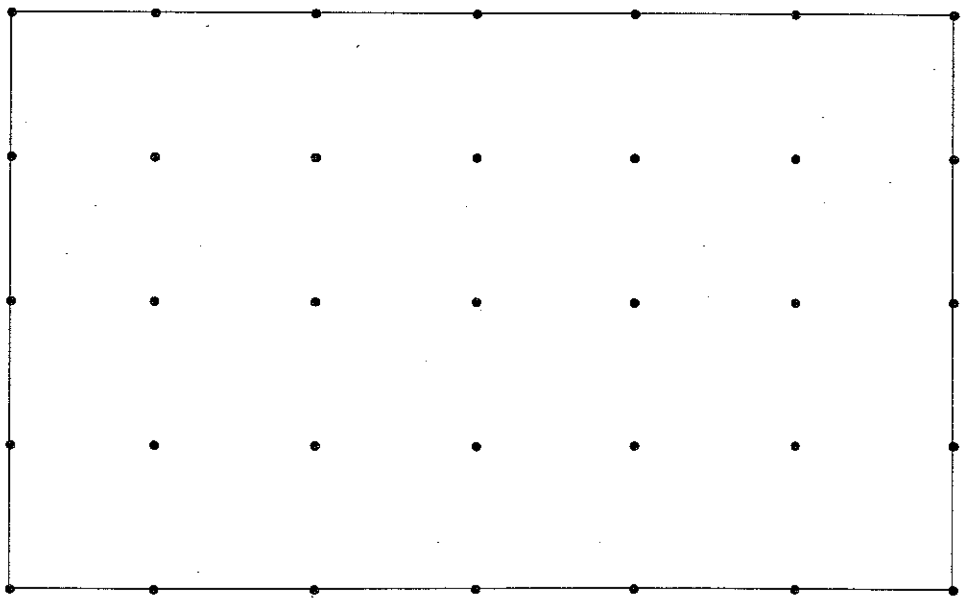


Name Date

Student Sheet 7

A Dot-Paper Square and Rectangle

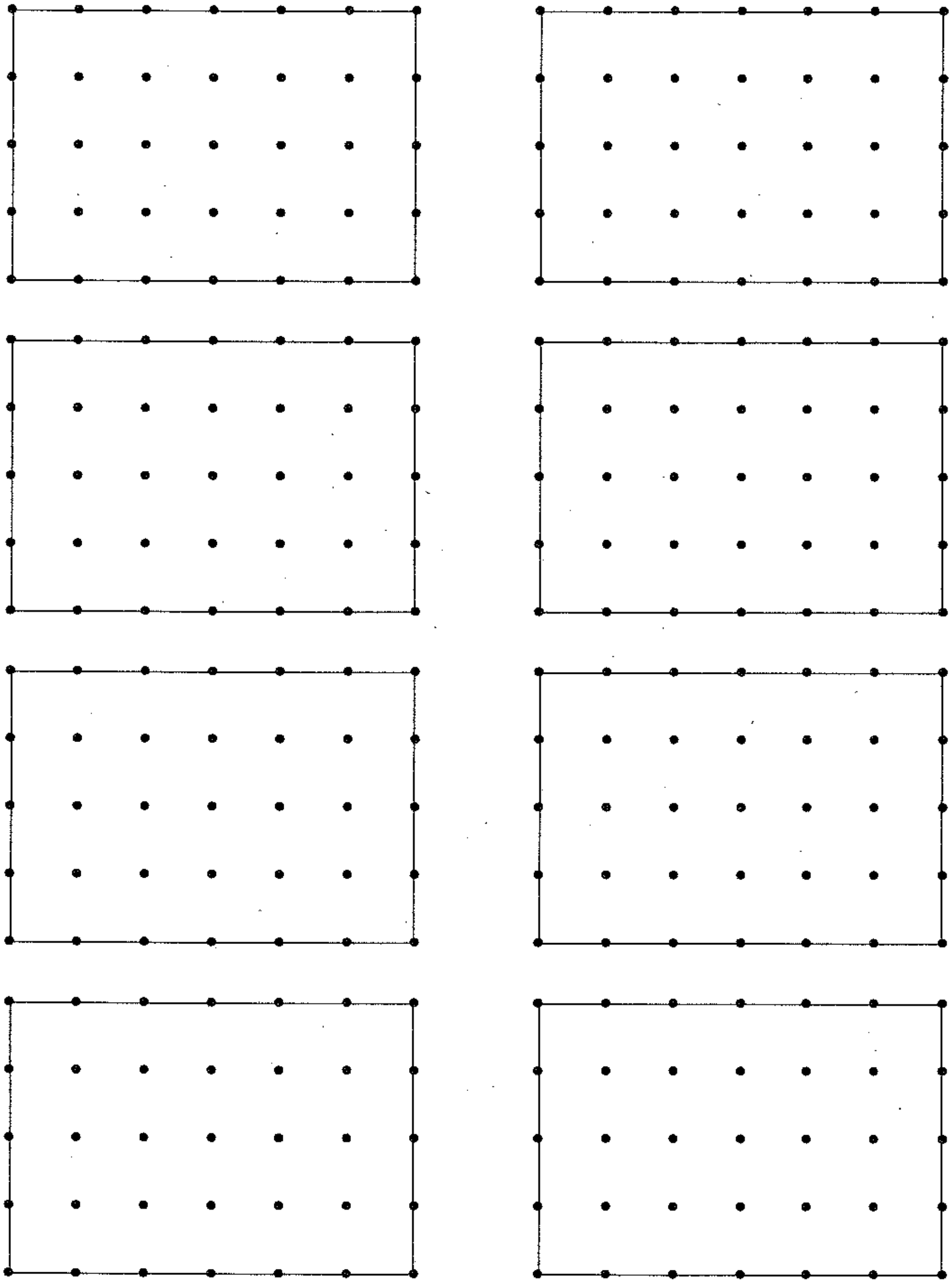




Name Date

Student Sheet 8

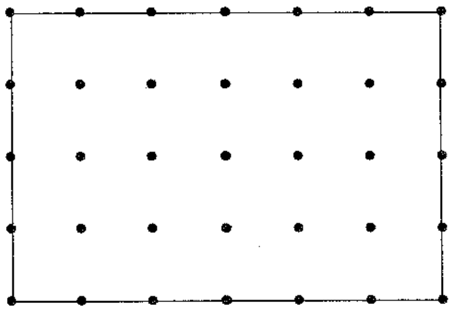
Dot-Paper Rectangles



Name ______________________ Date ______

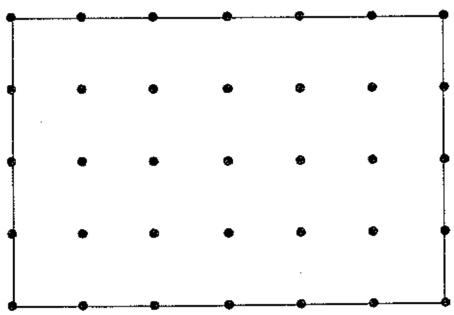
Thirds and Sixths

Divide this rectangle into thirds (3 equal parts).



Explain how you know the rectangle is divided into thirds.

Divide this rectangle into sixths (6 equal parts).



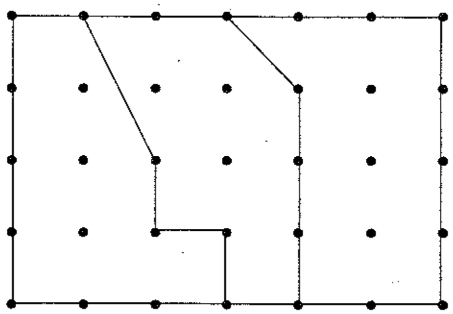
Explain how you know the rectangle is divided into sixths.

Name Date

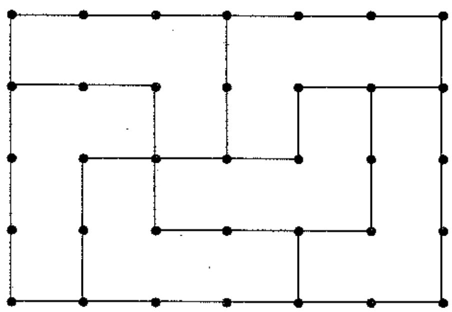
Student Sheet 10

Proving Thirds and Sixths

1. Is this rectangle divided into thirds? Explain your answer.



2. a. Is this rectangle divided into sixths? Explain your answer.



b. Color one-third of the rectangle in 2a. Explain how you know it is one-third.

3. Some students say $\frac{1}{6}$ is larger than $\frac{1}{4}$ because 6 is larger than 4. What do you think? Explain.